NAME ______________________ DATE ____________ CLASS ____________

Two-Dimensional Motion and Vectors

Problem A

FINDING RESULTANT MAGNITUDE AND DIRECTION

PROBLEM

A hummingbird flies 9.0 m horizontally and then flies up for 3.0 m. What is the bird's resultant displacement?

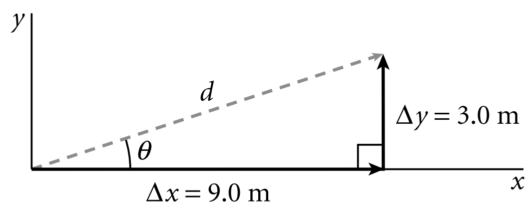
SOLUTION

1. DEFINE

Given: $\Delta x = 9.0$ m $\Delta y = 3.0$ m

Unknown: $d = ?$ $\theta = ?$

Diagram:



2. PLAN

Choose the equation(s) or situation: The Pythagorean theorem can be used to find the magnitude of the hummingbird's displacement. The direction of the displacement can be found using the tangent function.

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\tan\theta = \frac{\Delta y}{\Delta x}$$

Rearrange the equation(s) to isolate the unknown(s):

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$d = \sqrt{(9.0\text{ m})^2 + (3.0\text{ m})^2} = \sqrt{81\text{ m}^2 + 9.0\text{ m}^2} = \sqrt{9.0 \times 10^1\text{ m}^2}$$

$$d = \boxed{9.5\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{3.0\text{ m}}{9.0\text{ m}}\right)$$

$$\theta = \boxed{18° \text{ above horizonal}}$$

4. EVALUATE

The resultant displacement *(d)* is only slightly larger than the largest component (Δx), as is the case for small angles ($\theta \lesssim 20°$).

ADDITIONAL PRACTICE

1. A tiger paces back and forth along the front of its cage, which is 8 m wide. The tiger starts from the right side of the cage, paces to the left side, then back to the right side, and finally back to the left.
  a. What total distance has the tiger paced?
  b. What is the tiger's resultant displacement?

2. A particular type of rubber ball is able to bounce to 0.90 times the height from which it is dropped. The ball is dropped from a height of 0.91 m, but it falls slightly away from the vertical, so that by the time it has bounced to its new height it has undergone a horizontal displacement of 0.11 m. What is the ball's resultant displacement from its initial height to its maximum height after one bounce?

3. A helicopter flies 165 m horizontally and then moves downward to land 45 m below. What is the helicopter's resultant displacement?

4. A toy parachute is dropped from an open window that is 13.0 m above the ground. If the parachute travels 9.0 m horizontally, what is the resultant displacement?

5. An octopus swims 36.0 m east, 42.0 m north, and then rises 17.0 m toward the surface of the water. What is the octopus's displacement? (TWO-DIMENSIONAL METHOD: Visualize a horizontal and a vertical triangle. Find the horizontal resultant; use that with the vertical distance to calculate the final resultant. Studying this method can lead to understanding the easier three-dimensional solution in the solutions manual.)

6. An airplane taxis to the end of a runway before taking off. The magnitude of the plane's total displacement is 599 m. If the northern component of this displacement is 89 m, what is the displacement's eastern component? What is the direction of the total displacement?

7. The straightest stretch of railroad tracks in the world extends for 478 km in southwestern Australia. A train traveling along these tracks is displaced to the south by about 42 km. What is the train's displacement to the west? What is the direction of the total displacement?

8. Before the widespread use of steamships, sailing from Europe to North America was accomplished by use of the "trade winds." The trade winds move from the northeast to the southwest between 30° and 60° latitude in the northern hemisphere. A ship sailing from Europe to the Caribbean Sea would first travel southward to the Canary Islands, off the coast of North Africa, and then use the trade winds to sail west. Suppose a ship travels south from Iceland to the Canary Islands, and then west to Florida. The ship's total displacement is 7400 km at 26° south of west. If the ship sails 3200 km south from Iceland to the Canary Islands, how large is the western component of its journey?

9. The Palm Springs Aerial Tramway extends 3.88 km from the Valley Station, which is located 0.8 km above sea level, to the Mountain Station atop San Jacinto Peak. The actual path of the tramway's cables is not along a straight line, but if it were, the horizontal displacement of the tramway would be 3.45 km. How far is San Jacinto Peak above sea level?

10. The islands that form the Tristan da Cunha Group in the South Atlantic Ocean are considered to be the most remote places in the world: the next nearest inhabited island is 2400 km away. If you sail from Capetown, South Africa, in a south by southwest direction, you must travel 2.9×10^3 km before reaching the Tristan da Cunha islands. If the western component of your displacement is 2.8×10^3 km, what is your displacement south? In what direction is the resultant displacement?

Problem Bank Answers

Two-Dimensional Motion and Vectors

Additional Practice A

Givens | *Solutions*

1. $\Delta x_1 = 8$ m to the left $= +8$ m

$\Delta x_2 = 8$ m to the right $= -8$ m

$\Delta x_3 = 8$ m to the left $= +8$ m

a. distance traveled = 8 m + 8 m + 8 m = $\boxed{24 \text{ m}}$

b. $d = \Delta x_1 + \Delta x_2 + \Delta x_3 = 8 \text{ m} + (-8 \text{ m}) + 8 \text{ m} = \boxed{8 \text{ m}}$

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2. $h_i = 0.91$ m

$h_f = (0.90)h_i$

$\Delta x = 0.11$ m

$$\Delta y = h_f - h_i = (0.90 - 1.00)h_i$$

$$\Delta y = (-0.10)(0.91 \text{ m}) = -9.1 \times 10^{-2} \text{ m}$$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(0.11 \text{ m})^2 + (-9.1 \times 10^{-2} \text{ m})^2} = \sqrt{1.2 \times 10^{-2} \text{ m}^2 + 8.3 \times 10^{-3} \text{ m}^2}$$

$$d = 2.0 \times 10^{-2} \text{ m}^2 = \boxed{0.14 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{-9.1 \times 10^{-2} \text{ m}}{0.11 \text{ m}}\right)$$

$$\theta = -4.0 \times 10^{1\circ} = \boxed{4.0 \times 10^{1\circ} \text{ below the horizontal}}$$

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3. $\Delta x = 165$ m

$\Delta y = -45$ m

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(165 \text{ m})^2 + (-45 \text{ m})^2}\ \sqrt{2.72 \times 10^4 \text{ m}^2 + 2.0 \times 10^3 \text{ m}^2} = \sqrt{2.92 \times 10^4 \text{ m}^2}$$

$$d = \boxed{171 \text{ m}}$$

$$\theta \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{-45 \text{ m}}{165 \text{ m}}\right)$$

$$\theta = -15° = \boxed{15° \text{ below the horizontal}}$$

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4. $\Delta y = -13.0$ m

$\Delta x = 9.0$ m

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(9.0 \text{ m})^2 + (-13.0 \text{ m})^2} = \sqrt{81 \text{ m}^2 + 169 \text{ m}^2} = \sqrt{2.50 \times 10^2 \text{ m}^2}$$

$$d = \boxed{15.8 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)\ \tan^{-1}\left(\frac{-13.0 \text{ m}}{9.0 \text{ m}}\right)$$

$$\theta = -55° = \boxed{55° \text{ below the horizontal}}$$

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5. $\Delta x = 36.0$ m, east

$\Delta y = 42.0$ m, north

$\Delta z = 17.0$ m, up

$$d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} = \sqrt{(36.0 \text{ m})^2 + (42.0 \text{ m})^2 + (17.0 \text{ m})^2}$$

$$d = \sqrt{1.30 \times 10^3 \text{ m}^2 + 1.76 \times 10^3 \text{ m}^2 + 289 \text{ m}^2} = \sqrt{3.35 \times 10^3 \text{ m}^2}$$

$$d = \boxed{57.9 \text{ m}}$$

$$\text{horizontal direction} = \theta_h = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{42.0 \text{ m}}{36.0 \text{ m}}\right)$$

$$\theta_h = \boxed{49.4° \text{ north of east}}$$

$$\text{vertical direction} = \theta_v = \tan^{-1}\left(\frac{\Delta z}{\Delta x^2 + \Delta y^2}\right)$$

Givens	Solutions

$$\theta_v = \tan^{-1}\left(\frac{17.0\text{ m}}{\sqrt{(36.0\text{ m})^2 + (42.0\text{ m})^2}}\right) = \tan^{-1}\left(\frac{17.0\text{ m}}{\sqrt{1.30\times 10^3\text{ m}^2 + 1.76\times 10^3\text{ m}^2}}\right)$$

$$= \tan^{-1}\left(\frac{17.0\text{ m}}{\sqrt{3.06\times 10^3\text{ m}^2}}\right)$$

$$\theta_v = \boxed{17.1° \text{ above the horizontal}}$$

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6. $d = 599$ m

$\Delta y = 89$ m north

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\Delta x = \sqrt{d^2 - \Delta y^2} = \sqrt{(599\text{ m})^2 - (89\text{ m})^2} = \sqrt{3.59\times 10^5\text{ m}^2 - 7.9\times 10^3\text{ m}^2}$$

$$\Delta x = \sqrt{3.51\times 10^5\text{ m}^2}$$

$$\Delta x = \boxed{592\text{ m, east}}$$

$$\theta = \sin^{-1}\left(\frac{\Delta y}{d}\right) = \sin^{-1}\left(\frac{89\text{ m}}{599\text{ m}}\right)$$

$$\theta = \boxed{8.5\ ° \text{ north of east}}$$

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7. $d = 478$ km

$\Delta y = 42$ km, south $= -42$ km

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\Delta x = \sqrt{d^2 - \Delta y^2} = \sqrt{(478\text{ km})^2 - (-42\text{ km})^2}\ \sqrt{2.28\times 10^5\text{ km}^2 - 1.8\times 10^3\text{ km}^2}$$

$$\Delta x = \sqrt{2.26\times 10^5\text{ km}^2} = -475\text{ km}$$

$$\Delta x = \boxed{475\text{ km, west}}$$

$$\theta = \sin^{-1}\left(\frac{\Delta y}{d}\right) = \sin^{-1}\left(\frac{-42\text{ km}}{478\text{ km}}\right)$$

$$\theta = \boxed{5.0° \text{ south of west}}$$

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8. $d = 7400$ km

$\theta = 26°$ south of west

$\Delta y = 3200$ km, south $= -3200$ km

$$\text{d}^2 = \Delta x^2 = \Delta y^2$$

$$\Delta x = \sqrt{d^2 - \Delta y^2} = \sqrt{(7400\text{ km})^2 - (-3200\text{ km})^2} = \sqrt{5.5\times 10^7\text{ km}^2 - 1.0\times 10^7\text{ km}^2}$$

$$\Delta x = \sqrt{4.5\times 10^7\text{ km}^2} = -6700\text{ km}$$

$$\Delta x = \boxed{6700\text{ km, west}}$$

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9. $d = 3.88$ km

$\Delta x = 3.45$ km

$h_1 = 0.8$ km

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\Delta y = \sqrt{d^2 - \Delta x^2} = \sqrt{(3.88\text{ km})^2 - (3.45\text{ km})^2} = \sqrt{15.1\text{ km}^2 - 11.9\text{ km}^2} = \sqrt{3.2\text{ km}^2}$$

$$\Delta y = 1.8\text{ km}$$

height of mountain $= h = \Delta y + h_1 = 1.8\text{ km} + 0.8\text{ km}$

$$\boxed{h = 2.6\text{ km}}$$

Givens	*Solutions*
10. $d = 2.9 \times 10^3$ km $\Delta x = 2.8 \times 10^3$ km, west $= -2.8 \times 10^3$ km	$d^2 = \Delta x^2 + \Delta y^2$ $\Delta y = \sqrt{d^2 - \Delta x^2} = \sqrt{(2.9 \times 10^3 \text{ km})^2 - (-2.8 \times 10^3)^2}$ $\Delta y = \sqrt{8.4 \times 10^6 \text{ km}^2 - 7.8 \times 10^6 \text{ km}^2} = \sqrt{0.6 \times 10^6 \text{ km}^2} = -800$ km $\Delta y = \boxed{\text{800 km, south}}$ $\theta = \cos^{-1}\left(\frac{\Delta x}{d}\right) = \cos^{-1}\left(\frac{-2.8 \times 10^3 \text{ km}}{2.9 \times 10^3 \text{ km}}\right)$ $\theta = \boxed{15° \text{ south of west}}$

Additional Practice B

1. $d = 5.3$ km $\theta = 8.4°$ above horizontal	$\Delta y = d(\sin \theta) = (5.3 \text{ km})(\sin 8.4°)$ $\Delta y = 0.77 \text{ km} = 770$ m $\boxed{\text{the mountain's height} = 770 \text{ m}}$
2. $d = 19.1$ m $\theta = 3.0°$ to the left	$\Delta y = d(\sin \theta) = (19.1 \text{ m})(\sin 3.0°)$ $\Delta y = 1.0$ m to the left $\boxed{\text{the lane's width} = 1.0 \text{ m}}$
3. $d = 113$ m $\theta = 82.4°$ above the horizontal south	$\Delta x = d(\cos \theta) = (113 \text{ m})(\cos 82.4°)$ $\boxed{\Delta x = 14.9 \text{ m, south}}$
4. $v = 55$ km/h $\theta = 37°$ below the horizontal $= -37°$	$v_y = v(\sin \theta) = (55 \text{ km/h})[\sin(-37°)]$ $v_y = -33 \text{ km/h} = \boxed{\text{33 km/h, downward}}$
5. $d = 2.7$ m $\theta = 13°$ from the table's length	$\Delta x = d(\cos \theta) = (2.7 \text{ m})(\cos 13°)$ $\Delta x = \boxed{\text{2.6 m along the table's length}}$ $\Delta y = d(\sin \theta) = (2.7 \text{ m})(\sin 13°)$ $\Delta y = \boxed{\text{0.61 m along the table's width}}$
6. $v = 1.20$ m/s $\theta = 14.0°$ east of north	$v_x = v(\sin \theta) = (1.20 \text{ m/s})(\sin 14.0°)$ $v_x = \boxed{\text{0.290 m/s, east}}$ $v_y = v(\cos \theta) = (1.20 \text{ m/s})(\cos 14.0°)$ $v_y = \boxed{\text{1.16 m/s, north}}$
7. $d = 31.2$ km $\theta = 30.0°$ west of south	$\Delta x = d(\sin \theta) = (31.2 \text{ km})(\sin 30.0°)$ $\Delta x = \boxed{\text{15.6 km, west}}$ $\Delta y = d(\cos \theta) = (31.2 \text{ km})(\cos 30.0°)$ $\Delta y = \boxed{\text{27.0 km, south}}$