NAME ______________________ DATE ______________ CLASS ________________

Two-Dimensional Motion and Vectors

Problem B

RESOLVING VECTORS

PROBLEM

The straight stretch of Interstate Highway 5 from Mettler, California, to a point near Buttonwillow, California, is 53.0 km long and makes an angle of 48.7° north of west. What are the northern and western components of this highway segment?

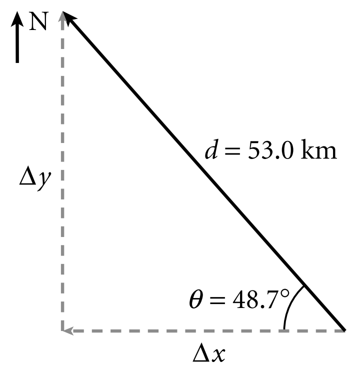
SOLUTION

1. DEFINE

Given: $d = 53.0$ km $\theta = 48.7°$ north of west

Unknown: $\Delta x = ?$ $\Delta y = ?$

Diagram:



2. PLAN

Choose the equation(s) or situation: Because the axes are perpendicular, the sine and cosine functions can be used to find the components.

$$\sin\theta = \frac{\Delta y}{d}$$

$$\cos\theta = \frac{\Delta x}{d}$$

Rearrange the equation(s) to isolate the unknown(s):

$$\Delta x = d(\cos\theta)$$

$$\Delta y = d(\sin\theta)$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\Delta x = (53.0\text{ km})(\cos 48.7°)$$

$$\Delta x = \boxed{35.0\text{ km, west}}$$

$$\Delta y = (53.0\text{ km})(\sin 48.7°)$$

$$\Delta y = \boxed{39.8\text{ km, north}}$$

4. EVALUATE

Using the Pythagorean theorem to check the answers confirms the magnitudes of the components.

$$d^2 = \Delta x^2 + \Delta y^2$$

$$(53.0\text{ km})^2 = (35.0\text{ km})^2 + (39.8\text{ km})^2$$

$$2.80 \times 10^3\ km^2 = 1.22 \times 10^3\ km^2 + 1.58 \times 10^3\ km^2$$

$$2.80 \times 10^3\ km^2 = 2.80 \times 10^3\ km^2$$

ADDITIONAL PRACTICE

1. The distance from an observer on the plain to the top of a nearby mountain is 5.3 km, and the angle between this line and the horizontal is 8.4°. How tall is the mountain?

2. A bowling ball is released at the near right corner of a bowling lane and travels 19.1 m at an angle of 3.0° with respect to the lane's length. The ball reaches the far left corner of the lane, where it knocks over the "7" pin. What is the width of the lane?

3. A skyrocket travels 113 m at an angle of 82.4° with respect to the ground and toward the south. What is the rocket's horizontal displacement?

4. A hot-air balloon descends with a velocity of 55 km/h at an angle of 37° below the horizontal. What is the vertical velocity of the balloon?

5. A billiard ball travels 2.7 m at an angle of 13° with respect to the long side of the table. What are the components of the ball's displacement?

6. One hole at a certain miniature golf course extends for about 60 m. A golf ball on this hole travels with a velocity of 1.20 m/s at 14.0° east of north. What are the eastern and northern components of the ball's velocity?

7. The Very Large Array in western New Mexico consists of several radio telescopes that can be rearranged along railroad tracks. The largest of these arrangements has the telescopes positioned in a "Y" pattern for 18 km along three separate tracks. Suppose an electrician inspects the instruments in each antenna from the end of the northern track to the end of the southwestern track. If the electrician's resultant displacement is 31.2 km at 30 0° west of south, what are the southern and western components of the displacement?

8. Barnard's Star is the closest star to Earth after the sun and the triple star Alpha Centauri. Barnard's Star has a velocity of 165.2 km/s at an angle of 32.7° away from its forward motion. What are the forward and side components of this velocity?

9. A tiger leaps with an initial velocity of 55.0 km/h at an angle of 13.0° with respect to the horizontal. What are the components of the tiger's velocity?

10. A certain type of balloon is designed to ascend rapidly. Suppose this balloon has a velocity 13.9 m/s at 26.0° above the horizontal and 24.0° east of north. What are the upward, northern, and eastern components of the balloon's velocity? (HINT: Draw horizontal and vertical right triangles whose sides represent the velocity's components.)

Givens | *Solutions*

10. $d = 2.9 \times 10^3$ km

$\Delta x = 2.8 \times 10^3$ km, west $= -2.8 \times 10^3$ km

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\Delta y = \sqrt{d^2 - \Delta x^2} = \sqrt{(2.9 \times 10^3 \text{ km})^2 - (-2.8 \times 10^3)^2}$$

$$\Delta y = \sqrt{8.4 \times 10^6 \text{ km}^2 - 7.8 \times 10^6 \text{ km}^2} = \sqrt{0.6 \times 10^6 \text{ km}^2} = -800 \text{ km}$$

$$\Delta y = \boxed{800 \text{ km, south}}$$

$$\theta = \cos^{-1}\left(\frac{\Delta x}{d}\right) = \cos^{-1}\left(\frac{-2.8 \times 10^3 \text{ km}}{2.9 \times 10^3 \text{ km}}\right)$$

$$\theta = \boxed{15° \text{ south of west}}$$

Additional Practice B

1. $d = 5.3$ km

$\theta = 8.4°$ above horizontal

$$\Delta y = d(\sin \theta) = (5.3 \text{ km})(\sin 8.4°)$$

$$\Delta y = 0.77 \text{ km} = 770 \text{ m}$$

$$\boxed{\text{the mountain's height} = 770 \text{ m}}$$

2. $d = 19.1$ m

$\theta = 3.0°$ to the left

$$\Delta y = d(\sin \theta) = (19.1 \text{ m})(\sin 3.0°)$$

$$\Delta y = 1.0 \text{ m to the left}$$

$$\boxed{\text{the lane's width} = 1.0 \text{ m}}$$

3. $d = 113$ m

$\theta = 82.4°$ above the horizontal south

$$\Delta x = d(\cos \theta) = (113 \text{ m})(\cos 82.4°)$$

$$\boxed{\Delta x = 14.9 \text{ m, south}}$$

4. $v = 55$ km/h

$\theta = 37°$ below the horizontal $= -37°$

$$v_y = v(\sin \theta) = (55 \text{ km/h})[\sin(-37°)]$$

$$v_y = -33 \text{ km/h} = \boxed{33 \text{ km/h, downward}}$$

5. $d = 2.7$ m

$\theta = 13°$ from the table's length

$$\Delta x = d(\cos \theta) = (2.7 \text{ m})(\cos 13°)$$

$$\Delta x = \boxed{2.6 \text{ m along the table's length}}$$

$$\Delta y = d(\sin \theta) = (2.7 \text{ m})(\sin 13°)$$

$$\Delta y = \boxed{0.61 \text{ m along the table's width}}$$

6. $v = 1.20$ m/s

$\theta = 14.0°$ east of north

$$v_x = v(\sin \theta) = (1.20 \text{ m/s})(\sin 14.0°)$$

$$v_x = \boxed{0.290 \text{ m/s, east}}$$

$$v_y = v(\cos \theta) = (1.20 \text{ m/s})(\cos 14.0°)$$

$$v_y = \boxed{1.16 \text{ m/s, north}}$$

7. $d = 31.2$ km

$\theta = 30.0°$ west of south

$$\Delta x = d\,(\sin \theta) = (31.2 \text{ km})(\sin 30.0°)$$

$$\Delta x = \boxed{15.6 \text{ km, west}}$$

$$\Delta y = d\,(\cos \theta) = (31.2 \text{ km})(\cos 30.0°)$$

$$\Delta y = \boxed{27.0 \text{ km, south}}$$

Givens	*Solutions*
8. $v = 165.2$ km/s $\theta = 32.7°$	$v_{forward} = v(\cos\theta) = (165.2\text{ km/s})(\cos 32.7°)$ $v_{forward} = \boxed{139\text{ km/s, forward}}$ $v_{side} = v(\sin\theta) = (165.2\text{ km/s})(\sin 32.7°)$ $v_{side} = \boxed{89.2\text{ km/s to the side}}$
9. $v = 55.0$ km/h $\theta = 13.0°$ above horizontal	$v_y = v(\sin\theta) = (55.0\text{ km/h})(\sin 13.0°)$ $v_y = \boxed{12.4\text{ km/h, upward}}$ $v_x = v(\cos\theta) = (55.0\text{ km/h})(\cos 13.0°)$ $v_x = \boxed{53.6\text{ km/h, forward}}$
10. $v = 13.9$ m/s $\theta_h = 24.0°$ east of north $\theta_v = 26.0°$ above the horizontal	$v_z = v(\sin\theta_v) = (13.9\text{ m/s})(\sin 26.0°)$ $v_z = \boxed{6.09\text{ m/s, upward}}$ horizontal velocity $= v_h = v(\cos\theta_v)$ $v_y = v_h(\cos\theta_h) = v(\cos\theta_v)(\cos\theta_h) = (13.9\text{ m/s})(\cos 26.0°)(\cos 24.0°)$ $v_y = \boxed{11.4\text{ m/s, north}}$ $v_x = v_h(\sin\theta_h) = v(\cos\theta_v)(\sin\theta_h) = (13.9\text{ m/s})(\cos 26.0°)(\sin 24.0°)$ $v_x = \boxed{5.08\text{ m/s, east}}$

Additional Practice C

1. $d_1 = 55$ km $\theta_1 = 37$ north of east $d_2 = 66$ km $\theta_2 = 0.0°$ (due east)	$\Delta x_1 = d_1(\cos\theta_1) = (55\text{ km})(\cos 37°) = 44\text{ km}$ $\Delta y_1 = d_1(\sin\theta_1) = (55\text{ km})(\sin 37°) = 33\text{ km}$ $\Delta x_2 = d_2(\cos\theta_2) = (66\text{ km})(\cos 0.0°) = 66\text{ km}$ $\Delta y_2 = d_2(\sin\theta_2) = (66\text{ km})(\sin 0.0°) = 0\text{ km}$ $\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 44\text{ km} + 66\text{ km} = 110\text{ km}$ $\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 33\text{ km} + 0\text{ km} = 33\text{ km}$ $d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(110\text{ km})^2 + (33\text{ km})^2}$ $= \sqrt{1.21 \times 10^4\text{ km}^2 + 1.1 \times 10^3\text{ km}^2} = \sqrt{1.32 \times 10^4\text{ km}^2}$ $d = \boxed{115\text{ km}}$ $\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{33\text{ km}}{110\text{ km}}\right)$ $\theta = \boxed{17°\text{ north of east}}$