

Two-Dimensional Motion and Vectors

Problem A**FINDING RESULTANT MAGNITUDE AND DIRECTION****PROBLEM**

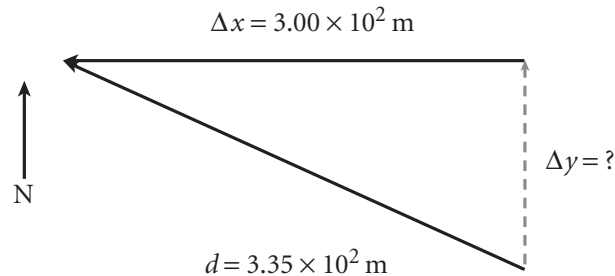
Cheetahs are, for short distances, the fastest land animals. In the course of a chase, cheetahs can also change direction very quickly. Suppose a cheetah runs straight north for 5.0 s, quickly turns, and runs 3.00×10^2 m west. If the magnitude of the cheetah's resultant displacement is 3.35×10^2 m, what is the cheetah's displacement and velocity during the first part of its run?

SOLUTION**1. DEFINE**

Given: $\Delta t_1 = 5.0$ s
 $\Delta x = 3.00 \times 10^2$ m
 $d = 3.35 \times 10^2$ m

Unknown: $\Delta y = ?$ $v_y = ?$

Diagram:

**2. PLAN**

Choose the equation(s) or situation: Use the Pythagorean theorem to subtract one of the displacements at right angles from the total displacement, and thus determine the unknown component of displacement.

$$d^2 = \Delta x^2 + \Delta y^2$$

Use the equation relating displacement to constant velocity and time, and use the calculated value for Δy and the given value for Δt to solve for v .

$$\Delta v = \frac{\Delta y}{\Delta t}$$

Rearrange the equation(s) to isolate the unknown(s):

$$\Delta y^2 = d^2 - \Delta x^2$$

$$\Delta y = \sqrt{d^2 - \Delta x^2}$$

$$v_y = \frac{\Delta y}{\Delta t}$$

3. CALCULATE

Substitute the values into the equation(s) and solve: Because the value for Δy is a displacement magnitude, only the positive root is used ($\Delta y > 0$).

$$\begin{aligned} \Delta y &= \sqrt{(3.35 \times 10^2 \text{ m})^2 - (3.00 \times 10^2 \text{ m})^2} \\ &= \sqrt{1.12 \times 10^5 \text{ m}^2 - 9.00 \times 10^4 \text{ m}^2} \end{aligned}$$

$$= \sqrt{2.2 \times 10^4 \text{ m}}$$

$$= \boxed{1.5 \times 10^2 \text{ m, north}}$$

$$v_y = \frac{1.5 \times 10^2 \text{ m}}{5.0 \text{ s}} = \boxed{3.0 \times 10^1 \text{ m/s, north}}$$

4. EVALUATE

The cheetah has a top speed of 30 m/s, or 107 km/h. This is equal to about 67 miles/h.

ADDITIONAL PRACTICE

- An ostrich cannot fly, but it is able to run fast. Suppose an ostrich runs east for 7.95 s and then runs 161 m south, so that the magnitude of the ostrich's resultant displacement is 226 m. Calculate the magnitude of the ostrich's eastward component and its running speed.
- The pronghorn antelope, found in North America, is the best long-distance runner among mammals. It has been observed to travel at an average speed of more than 55 km/h over a distance of 6.0 km. Suppose the antelope runs a distance of 5.0 km in a direction 11.5° north of east, turns, and then runs 1.0 km south. Calculate the resultant displacement.
- Kangaroos can easily jump as far 8.0 m. If a kangaroo makes five such jumps westward, how many jumps must it make northward to have a northwest displacement with a magnitude of 68 m? What is the angle of the resultant displacement with respect to north?
- In 1926, Gertrude Ederle of the United States became the first woman to swim across the English channel. Suppose Ederle swam 25.2 km east from the coast near Dover, England, then made a 90° turn and traveled south for 21.3 km to a point east of Calais, France. What was Ederle's resultant displacement?
- The emperor penguin is the best diver among birds: the record dive is 483 m. Suppose an emperor penguin dives vertically to a depth of 483 m and then swims horizontally a distance of 225 m. What angle would the vector of the resultant displacement make with the water's surface? What is the magnitude of the penguin's resultant displacement?
- A killer whale can swim as fast as 15 m/s. Suppose a killer whale swims in one direction at this speed for 8.0 s, makes a 90° turn, and continues swimming in the new direction with the same speed as before. After a certain time interval, the magnitude of the resultant displacement is 180.0 m. Calculate the amount of time the whale swims after changing direction.
- Woodcocks are the slowest birds: their average speed during courtship displays can be as low as 8.00 km/h. Suppose a woodcock flies east for 15.0 min. It then turns and flies north for 22.0 min. Calculate the magnitude of the resultant displacement and the angle between the resultant displacement and the woodcock's initial displacement.

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Additional Practice A

Givens

1. $\Delta t_x = 7.95 \text{ s}$

$\Delta y = 161 \text{ m}$

$d = 226 \text{ m}$

Solutions

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\Delta x = \sqrt{d^2 - \Delta y^2} = \sqrt{(226 \text{ m})^2 - (161 \text{ m})^2} = \sqrt{5.11 \times 10^4 \text{ m}^2 - 2.59 \times 10^4 \text{ m}^2}$$

$$\Delta x = \sqrt{2.52 \times 10^4 \text{ m}^2} = 159 \text{ m}$$

$$\Delta x = \boxed{159 \text{ m}}$$

$$v = \frac{\Delta x}{\Delta t_x} = \frac{159 \text{ m}}{7.95 \text{ s}} = \boxed{20.0 \text{ m/s}}$$

2. $d_1 = 5.0 \text{ km}$

$\theta_1 = 11.5^\circ$

$d^2 = 1.0 \text{ km}$

$\theta_2 = -90.0^\circ$

$$\Delta x_{tot} = d_1(\cos \theta_1) + d_2(\cos \theta_2) = (5.0 \text{ km})(\cos 11.5^\circ) + (1.0 \text{ km})[\cos(-90.0^\circ)]$$

$$\Delta x_{tot} = 4.9 \text{ km}$$

$$\Delta y_{tot} = d_1(\sin \theta_1) + d_2(\sin \theta_2) = (5.0 \text{ km})(\sin 11.5^\circ) + (1.0 \text{ km})[\sin(-90.0^\circ)]$$

$$= 1.0 \text{ km} - 1.0 \text{ km}$$

$$\Delta y_{tot} = 0.0 \text{ km}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(4.9 \text{ km})^2 + (0.0 \text{ km})^2}$$

$$d = \boxed{4.9 \text{ km}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta y_{tot}}{\Delta x_{tot}} \right) = \tan^{-1} \left(\frac{0.0 \text{ km}}{4.9 \text{ km}} \right) = \boxed{0.0^\circ, \text{ or due east}}$$

3. $\Delta x = 5 \text{ jumps}$

1 jump = 8.0 m

$d = 68 \text{ m}$

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\Delta y = \sqrt{d^2 - \Delta x^2} = \sqrt{(68 \text{ m})^2 - [(5)(8.0 \text{ m})]^2} = \sqrt{4.6 \times 10^3 \text{ m}^2 - 1.6 \times 10^3 \text{ m}^2}$$

$$\Delta y = \sqrt{3.0 \times 10^3 \text{ m}^2} = 55 \text{ m}$$

$$\text{number of jumps northward} = \frac{55 \text{ m}}{8.0 \text{ m/jump}} = 6.9 \text{ jumps} = \boxed{7 \text{ jumps}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta x}{\Delta y} \right) = \tan^{-1} \left[\frac{(5)(8.0 \text{ m})}{55 \text{ m}} \right] = \boxed{36^\circ \text{ west of north}}$$

4. $\Delta x = 25.2 \text{ km}$

$\Delta y = 21.3 \text{ km}$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(25.2 \text{ km})^2 + (21.3 \text{ km})^2}$$

$$d = \sqrt{635 \text{ km}^2 + 454 \text{ km}^2} = \sqrt{1089 \text{ km}^2}$$

$$d = \boxed{33.00 \text{ km}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left(\frac{21.3 \text{ km}}{25.2 \text{ km}} \right)$$

$$\theta = \boxed{42.6^\circ \text{ south of east}}$$

Givens

5. $\Delta y = -483 \text{ m}$
 $\Delta x = 225 \text{ m}$

Solutions

$$\theta = \tan^{-1} \left(\frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left(\frac{-483}{225} \right) = -65.0^\circ = \boxed{65.0^\circ \text{ below the waters surface}}$$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(225 \text{ m})^2 + (-483 \text{ m})^2}$$

$$d = \sqrt{5.06 \times 10^4 \text{ m}^2 + 2.33 \times 10^5 \text{ m}^2} = \sqrt{2.84 \times 10^5 \text{ m}^2}$$

$$d = \boxed{533 \text{ m}}$$

6. $v = 15.0 \text{ m/s}$
 $\Delta t_x = 8.0 \text{ s}$
 $d = 180.0 \text{ m}$

$$d^2 = \Delta x^2 + \Delta y^2 = (v\Delta t_x)^2 + (v\Delta t_y)^2$$

$$d^2 = v^2(\Delta t_x^2 + \Delta t_y^2)$$

$$\Delta t_y = \sqrt{\left(\frac{d}{v}\right)^2 - \Delta t_x^2} = \sqrt{\left(\frac{180.0 \text{ m}}{15.0 \text{ m/s}}\right)^2 - (8.0 \text{ s})^2} = \sqrt{144 \text{ s}^2 - 64 \text{ s}^2} = \sqrt{8.0 \times 10^1 \text{ s}^2}$$

$$\Delta t_y = \boxed{8.9 \text{ s}}$$

7. $v = 8.00 \text{ km/h}$
 $\Delta t_x = 15.0 \text{ min}$
 $\Delta t_y = 22.0 \text{ min}$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(v\Delta t_x)^2 + (v\Delta t_y)^2}$$

$$= v\sqrt{\Delta t_x^2 + \Delta t_y^2}$$

$$d = (8.00 \text{ km/h}) \left(\frac{1 \text{ h}}{60 \text{ min}} \right) \sqrt{(15.0 \text{ min})^2 + (22.0 \text{ min})^2}$$

$$d = (8.00 \text{ km/h}) \left(\frac{1 \text{ h}}{60 \text{ min}} \right) \sqrt{225 \text{ min}^2 + 484 \text{ min}^2}$$

$$d = \left(\frac{8.00 \text{ km}}{60 \text{ min}} \right) \sqrt{709 \text{ min}^2} = \boxed{3.55 \text{ km}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left(\frac{v\Delta t_y}{v\Delta t_x} \right) = \tan^{-1} \left(\frac{\Delta t_y}{\Delta t_x} \right) = \tan^{-1} \left(\frac{22.0 \text{ min}}{15.0 \text{ min}} \right)$$

$$\theta = \boxed{55.7^\circ \text{ north of east}}$$

Additional Practice B

1. $d = (5)(33.0 \text{ cm})$
 $\Delta y = 88.0 \text{ cm}$

$$\theta = \sin^{-1} \left(\frac{\Delta y}{d} \right) = \sin^{-1} \left[\frac{88.0 \text{ cm}}{(5)(33.0 \text{ cm})} \right] = \boxed{32.2^\circ \text{ north of west}}$$

$$\Delta x = d(\cos \theta) = (5)(33.0 \text{ cm})(\cos 32.2^\circ) = \boxed{1.40 \times 10^2 \text{ cm to the west}}$$

2. $\theta = 60.0^\circ$
 $d = 10.0 \text{ m}$

$$\Delta x = d(\cos \theta) = (10.0 \text{ m})(\cos 60.0^\circ) = \boxed{5.00 \text{ m}}$$

$$\Delta y = d(\sin \theta) = (10.0 \text{ m})(\sin 60.0^\circ) = \boxed{8.66 \text{ m}}$$

3. $d = 10.3 \text{ m}$
 $\Delta y = -6.10 \text{ m}$

Finding the angle between d and the x -axis yields,

$$\theta_1 = \sin^{-1} \left(\frac{\Delta y}{d} \right) = \sin^{-1} \left(\frac{-6.10 \text{ m}}{10.3 \text{ m}} \right) = -36.3^\circ$$

The angle between d and the negative y -axis is therefore,

$$\theta = -90.0 - (-36.3^\circ) = -53.7^\circ$$

$$\theta = \boxed{53.7^\circ \text{ on either side of the negative } y\text{-axis}}$$

$$d^2 + \Delta x^2 + \Delta y^2$$

$$\Delta x = \sqrt{d^2 - \Delta y^2} = \sqrt{(10.3 \text{ m})^2 - (-6.10 \text{ m})^2} = \sqrt{106 \text{ m}^2 - 37.2 \text{ m}^2} = \sqrt{69 \text{ m}^2}$$

$$\Delta x = \boxed{\pm 8.3 \text{ m}}$$